

Some simple gamma matrix algebra.

```
::PostDefaultRules( @@prodsort!(%), @@eliminate_kr!(%),  
                    @@canonicalise!(%), @@collect_terms!(%) ).
```

Assigning property PostDefaultRules to .

```
{s,r,l,k,m,n}::Indices(vector).  
{s,r,l,k,m,n}::Integer(0..d-1).  
\Gamma_{#}::GammaMatrix(metric=\delta).  
\delta_{m n}::KroneckerDelta.
```

Assigning property Indices to s, r, l, k, m, n.

Assigning property Integer to s, r, l, k, m, n.

Assigning property GammaMatrix to \Gamma.

Assigning property KroneckerDelta to \delta.

The expression which we want to simplify:

```
\Gamma_{s r} \Gamma_{r l} \Gamma_{k m} \Gamma_{m s};
```

$$1 := (-1) \Gamma_{mr} \Gamma_{lm} \Gamma_{ks} \Gamma_{rs};$$

By joining all adjacent gamma matrices repeatedly (using the !! form of the command), we end up with

```
@join!!(%) {expand};
```

$$1 := (-1) (2 \Gamma_{lr} - \Gamma_{lr} d + \delta_{lr} d - \delta_{lr}) (2 \Gamma_{kr} - \Gamma_{kr} d + \delta_{kr} - \delta_{kr} d);$$

After distributing the result and joining once more, we collect factors and get the desired result:

```
@distribute!(%):  
@canonicalise!(%):  
@join!(%) {expand}:  
@distribute!(%):  
@factorise!(%) {d}:  
@collect_factors!(%);
```

$$1 := \Gamma_{kl} (12 - 18 d + 8 d^2 - d^3) + \delta_{kl} (-3 + 6 d - 4 d^2 + d^3);$$